APPROXIMATE FORMULAS FOR THE VISCOSITY AND HEAT CONDUCTION COEFFICIENTS OF PARTIALLY DISSOCIATED AND IONIZED AIR

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A simple and sufficiently accurate computation of the transport coefficients (the viscosity and heat conductivity coefficients for gas and plasma mixtures) in a broad range of temperatures and pressures acquires practical value in the solution of physicochemical hydrodynamics problems, particularly, supersonic aerodynamics and heat transfer problems. The fact is that the formulas of the strict kinetic theory of gases for the transfer coefficients [1-3] derived in the asymptotic limit of small Knudsen numbers remain sufficiently complicated for the solution of specific hydrodynamic problems despite their substantial simplification obtained in [2]. The formulas are represented in the form of the ratio of determinants of order $N\xi$, where N is the number of mixture components and ξ is the number of the approximation (the number of terms in the expansions of the desired transfer coefficients in Sonine polynomials [2]). This results in the fact that for the solution of flow problems, say, computation of the transfer coefficients by using these formulas, occupies a large portion of the time than the time for a numerical solution of the differential equations themselves [4]. Consequently, in practice all computations of transfer coefficients in multicomponent gas and plasma mixture flow problems are performed by using different approximate formulas. Computations of just some transfer coefficients by the strict formulas of kinetic gas theory in the first approximations different from zero [1] for constant given element concentrations without the solution of any problems [5] were performed for the viscosity and heat conductivity of a H_2—He mixture (Jupiter atmosphere), for the viscosity and heat conductivity of air (earth's atmosphere) in [6], for the viscosity of a CO_2—N_2 mixture (Venus atmosphere) in [7]. All the equilibrium transfer coefficients for partially dissociated and ionized air were first calculated in [8] by means of formulas [2, 9] up to the fourth approximation inclusive.

Evaluation of the transfer coefficients by the exact formulas of kinetic gas theory is associated with the necessity to know the collision integrals between the different mixture components. Each interaction requires knowledge of the potential interaction function, and $(1/2)N(N + 1)$ potential functions are required for an N-component mixture. The existing spread in data on particle interaction potentials [10, 11] in the area of ionization development results in uncertainty in the value of the effective equilibrium heat conductivity coefficient of up to 30%, and the uncertainty in the values of the viscosity coefficient is of the same order [12, 13]. At the same time the error in the transfer coefficients has slight influence on the accuracy of the calculation of the integral aerodynamic and thermal characteristics. Thus, the uncertainty in the transfer coefficients in the solution of supersonic flow problems around a sphere by air in chemical and ionization equilibrium within the framework of the complete viscous shock layer equations results in an uncertainty 5-10 times less in the value of the convective heat flux to the body [13]. Therefore, there is a foundation to assume that replacement of the exact by approximate computation in evaluation of the transfer coefficients will not result in substantial uncertainty in the integral flow characteristics. All the above does not remove the doubts about the expediency and practical necessity of obtaining simple approximate formulas for the transfer formulas of gas and plasma mixtures.

A whole number of approximate formulas have been proposed in the literature for the viscosity [13-19] and heat conductivity [19-24] coefficients. Most extensively utilized in heat transfer problems is the Wilke formula [16] for the viscosity and the Mason and Sexena formula [21] for the thermal conductivity. However, the accuracy of these formulas becomes unsatisfactory upon the appearance of ionized components in the mixture [25]. In this connection, new approximate formulas for the viscosity and heat conductivity coefficients are proposed in [26, 27] on the basis of numerical computations [6], that improve the formulas

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[16, 21] in the ionization domain but yields noticeable error as compared with the data of [6] which differ, in turn, from the exact data [8] by approximately 15% for the viscosity and ~60% for the thermal conductivity.

On the basis of expressions for the viscosity and thermal conductivity coefficients in the first nonzero approximations [see (1.1), and (1.2)] in which nondiagonal elements are later discarded, and exact expressions for the ionic viscosity and electronic thermal conductivity within the full ionization limit, approximate formulas are derived in this paper for the viscosity and transport thermal conductivity coefficients (due to translational degrees of freedom of a components) for partially dissociated and ionized air that yields good agreement with the results of exact computations with higher approximations taken into account [8].

1. Let us consider the exact expressions for the viscosity and transport thermal conductivity coefficients in the first nonzero approximation (for seeking transfer coefficients in the form of series in Sonine polynomials) [1]:

$$\mu(1) = -\left|\begin{matrix} 0 & x_1 & \dots & x_N \\ x_1 & H_{11} & \dots & H_{1N} \\ x_2 & H_{21} & \dots & H_{2N} \\ \vdots & & & \vdots \\ x_N & H_{N1} & \dots & H_{NN} \end{matrix}\right| \Bigg/ \left|\begin{matrix} H_{11} & H_{12} & \dots & H_{1N} \\ & & & \\ H_{21} & H_{22} & \dots & H_{2N} \\ \vdots & & & \vdots \\ H_{N1} & H_{N2} & \dots & H_{NN} \end{matrix}\right|; \tag{1.1}$$

$$\lambda(2) = 4\left|\begin{matrix} 0 & x_1 & \dots & x_N \\ x_1 & L_{11} & \dots & L_{1N} \\ x_2 & L_{21} & \dots & L_{2N} \\ \vdots & & & \vdots \\ x_N & L_{N1} & \dots & L_{NN} \end{matrix}\right| \Bigg/ \left|\begin{matrix} L_{11} & L_{12} & \dots & L_{1N} \\ L_{21} & L_{22} & \dots & L_{2N} \\ \vdots & & & \vdots \\ L_{N1} & L_{N2} & \dots & L_{NN} \end{matrix}\right|, \tag{1.2}$$

where

$$H_{ii} = \frac{x_i^2}{\mu_i} + \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{2x_i x_k}{\mu_{ik}} \frac{m_i m_k}{(m_i + m_k)^2} \left(\frac{5}{3A_{ik}^*} + \frac{m_k}{m_i}\right); \tag{1.3}$$

$$H_{ij} = -\frac{2x_i x_j}{\mu_{ij}} \frac{m_i m_j}{(m_i + m_j)^2} \left(\frac{5}{3A_{ij}^*} - 1\right) \qquad (i \neq j); \tag{1.4}$$

$$L_{ii} = -\frac{4x_i^2}{\lambda_i} - \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{2x_i x_k}{\lambda_{ik}} \frac{\frac{15}{2} m_i^2 + \frac{25}{4} m_k^2 - 3m_k^2 B_{ik}^* + 4m_i m_k A_{ik}^*}{(m_i + m_k)^2 A_{ik}^*}; \tag{1.5}$$

$$L_{ij} = \frac{2x_i x_j}{\lambda_{ij}} \frac{m_i m_j}{(m_i + m_j)^2} \left(\frac{55}{4A_{ij}^*} - 3\frac{B_{ij}^*}{A_{ij}^*} - 4\right) \qquad (i \neq j); \tag{1.6}$$

$$\mu_i = \frac{8.387 \cdot 10^{-6} \sqrt{T m_i}}{Q_{ii}^{(2,2)}}, \; \mu_{ij} = \frac{8.387 \cdot 10^{-6}}{Q_{ij}^{(2,2)}} \sqrt{\frac{2m_i m_j T}{m_i + m_j}}; \tag{1.7}$$

$$\lambda_i = \frac{0.2615 \sqrt{T/m_i}}{Q_{ii}^{(2,2)}}, \; \lambda_{ij} = \frac{0.2615}{Q_{ij}^{(2,2)}} \sqrt{\frac{(m_i + m_j) T}{2m_i m_j}}; \tag{1.8}$$

$$Q_{ij}^{(l,s)} = \pi\sigma_{ij}^2 \Omega_{ij}^{(l,s)*}; \tag{1.9}$$

$$A_{ij}^* = \frac{\Omega_{ij}^{(2,2)*}}{\Omega_{ij}^{(1,1)*}} = \frac{Q_{ij}^{(2,2)}}{Q_{ij}^{(1,1)}}; \tag{1.10}$$

$$B_{ij}^* = \frac{5\Omega_{ij}^{(1,2)*} - 4\Omega_{ij}^{(1,3)*}}{\Omega_{ij}^{(1,1)*}}. \tag{1.11}$$

Here m_i and x_i are the molar mass and concentration of the components, μ_i (Pa·sec) and λ_i (W/mK) are the viscosity and thermal conductivity coefficients of the pure i-th component; μ_{ij} (Pa·sec) and λ_{ij} (W/mK) are the auxiliary viscosity and thermal conductivity coefficients that are analogs of the binary diffusion coefficients, $\Omega_{ij}^{(l,s)*}$ are relative collision integrals of order (ℓ, s) governing the distinction between some particle interaction model and the idealized solid sphere model [1], and σ_{ij} is the diameter of the gaskinetic i − j collision.

For mixtures of neutral monatomic gases with molar masses of the same order, taking account of just the lower expansions in the Sonine polynomial expansions at temperatures above room and below dissociation temperatures results, as a rule, in not more than a ~0.3% error for the viscosity coefficient and a ~0.5% error for the thermal conductivity coefficient.

As follows from computations by the exact formulas of kinetic gas theory with higher approximations taken into account for the dissociated and partially ionized air ($x_E \lesssim 0.4$), the viscosity coefficient $\mu(\xi)$ can be calculated with an ~1% error in a first approximation ($\xi = 1$). For strongly ionized air $x_E > 0.4$) the viscosity coefficient computation in a first approximation results in up to ~15% error, however, the second approximation already turns out to be sufficiently exact, a further increase in ξ has no effect on the accuracy of computing μ within ~1% limits [8].

Values of the thermal conductivity coefficient $\lambda(\xi)$ of partially dissociated and ionized air, computed in the second, third, and fourth approximations for a $p = 10^3$-10^7 Pa·sec pressure and temperatures 5000 to 20,000 K in [8] show that the second approximation (i.e., the first nonzero approximation) has ~1% accuracy in the low temperature domain ($T \lesssim 6000$ K) while the thermal conductivity coefficient, calculated in the second approximation ($\xi = 2$), differs from that computed in the third approximation by approximately 60%. The fourth approximation refines the third by ~1-2%. Consequently the thermal conductivity coefficient must be computed in at least the third (second nonzero) approximation.

However, when obtaining approximate formulas for the transfer coefficients that agree with the exact computations in higher approximations, expressions in the lowest nonzero approximations and limit formulas for full ionization are taken as basis, the corrections introduced by the higher approximations are taken into account by extracting weakly varying quantities in the initial expressions and replacing them by effective coefficients selected empirically from a comparison with the exact computations in appropriate approximations.

2. We start from (1.1) and (1.2), which can be represented in the form of the series [1]

$$\mu = \sum_{i=1}^{N} \frac{x_i^2}{H_{ii}} - \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{x_i x_j H_{ij}}{H_{ii} H_{jj}} + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\substack{k=1 \\ i \neq k \neq j}}^{N} \frac{x_i x_j H_{ij} H_{ik}}{H_{ii} H_{jj} H_{kk}} - \ldots; \tag{2.1}$$

$$\lambda = -4 \sum_{i=1}^{N} \frac{x_i^2}{L_{ii}} + 4 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{x_i x_j L_{ij}}{L_{ii} L_{jj}} - 4 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sum_{\substack{k=1 \\ i \neq k \neq j}}^{N} \frac{x_i x_j L_{ij} L_{ik}}{L_{ii} L_{jj} L_{kk}} + \cdots \tag{2.2}$$

to obtain approximate formulas for u and λ. Detailed computations [5, 28] showed that $H_{ij} \ll H_{ii}$ and $L_{ij} \ll L_{ii}$ ($i \neq j$). Consequently, we take

$$\mu = \sum_{i=1}^{N} \frac{x_i^2}{H_{ii}}; \tag{2.3}$$

$$\lambda = -4 \sum_{i=1}^{N} \frac{x_i^2}{L_{ii}} \tag{2.4}$$

as initial expressions to find the approximate formulas.

It is seen from the computations that the functions A_{ij}^* in (1.10) depend weakly on the temperature but are generally different for different kinds of interactions. The functions B_{ij}^* in (1.11) also depend weakly on the temperature. It follows from the analysis of these coefficients that in practice $1 \lesssim B_{ij}^* \lesssim 1.5$ within a broad range of values of the pressure and temperature for all kinds of interactions (the collision integrals were taken from [10, 11]). The value $B_{ij}^* = 1.25$ was used in [27] for all i, j pairs. In this case the expressions for L_{ij} and L_{ii} simplify

$$L_{ij} = 2 x_i x_j m_i m_j \frac{10 - 4A_{ij}^*}{(m_i + m_j)^2 \lambda_{ij} A_{ij}^*} \quad (i \neq j); \tag{2.5}$$

$$L_{ii} = -4 \frac{x_i^2}{\lambda_i} - \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{2 x_i x_k \left[\frac{15}{2} m_i^2 + \frac{5}{2} m_k^2 + 4 m_i m_k A_{ik}^*\right]}{(m_i + m_k)^2 \lambda_{ik} A_{ik}^*}. \tag{2.6}$$

The functions $Q_{ij}^{(2.2)}$ depend strongly on the temperature and the (i, j) pairs of interacting components. Consequently, in [26] it is proposed to express σ_{ij} in terms of the collision diameter and the viscosity coefficients μ_i (or thermal conductivity λ_i) of pure gases

$$\sigma_{ij}^2 = \frac{1}{4}(\sigma_i + \sigma_j)^2 = C_1 \frac{\sqrt{T}}{4} \left[\left(\frac{m_i^{1/2}}{\mu_i \Omega_{ii}^{(2,2)*}} \right)^{1/2} + \left(\frac{m_j^{1/2}}{\mu_j \Omega_{jj}^{(2,2)*}} \right)^{1/2} \right]^2 = C_2 \frac{\sqrt{T}}{4} \left[\left(\frac{m_i^{-1/2}}{\lambda_i \Omega_{ii}^{(2,2)*}} \right)^{1/2} + \left(\frac{m_j^{-1/2}}{\lambda_j \Omega_{jj}^{(2,2)*}} \right)^{1/2} \right]^2 \tag{2.7}$$

(C_1, C_2 are constants dependent on the dimensionality of the quantities). Substituting (2.7) into (1.9), and then (1.9) into the expressions for μ_{ij} and λ_{ij}, we obtain

$$\mu_{ij}=\sqrt{\frac{32m_im_j}{m_i+m_j}}\left[\frac{m_i^{1/4}}{\mu_i^{1/2}}F_{ij}+\frac{m_j^{1/4}}{\mu_j^{1/2}}B_{ij}\right]^2; \tag{2.8}$$

$$\lambda_{ij}=\sqrt{8\frac{m_i+m_j}{m_im_j}}\left[\frac{m_i^{-1/4}}{\lambda_i^{1/2}}F_{ij}+\frac{m_j^{-1/4}}{\lambda_j^{1/2}}B_{ij}\right]^2, \tag{2.9}$$

where

$$F_{ij}=\sqrt{\frac{Q_{ij}^{(2,2)}}{Q_{ii}^{(2,2)}}},\ B_{ij}=\sqrt{\frac{Q_{ij}^{(2,2)}}{Q_{jj}^{(2,2)}}}. \tag{2.10}$$

The advantage of (2.8) and (2.9) over the formulas (1.7) and (1.8) for μ_{ij} and λ_{ij} is that F_{ij} and B_{ij} are weakly varying functions of the temperature and can be set equal to constants for the majority of interaction pairs. Substituting (2.8) and (2.9) into (2.3) and (2.4), respectively, we find

$$\mu=\sum_{i=1}^{N}\frac{x_i\mu_i}{x_i+\sum\limits_{\substack{j=1\\ j\neq i}}^{N}x_j\frac{\frac{5}{3}m_i/A_{ij}^*+m_j}{m_i+m_j}\Phi_{ij}}; \tag{2.11}$$

$$\lambda=\sum_{i=1}^{N}\frac{x_i\lambda_i}{x_i+\sum\limits_{\substack{j=1\\ j\neq i}}^{N}x_j\frac{\frac{15}{4}m_i^2+\frac{5}{4}m_j^2+2m_im_jA_{ij}^*}{(m_i+m_j)^2A_{ij}^*}\Phi_{ij}}. \tag{2.12}$$

Here

$$\Phi_{ij}=\frac{\left[F_{ij}+\sqrt{\mu_i/\mu_j}\,(m_j/m_i)^{1/4}B_{ij}\right]^2}{2\sqrt{2}\sqrt{1+m_i/m_j}},\ \frac{\mu_i}{\mu_j}\sqrt{\frac{m_j}{m_i}}=\frac{\lambda_i}{\lambda_j}\sqrt{\frac{m_i}{m_j}}. \tag{2.13}$$

For interactions of neutral atoms and molecules $A_{ij}^* = 5/3$. Consequently, if we set $A_{ij}^* = 5/3$ and $F_{ij} = B_{ij} = 1$, then (2.11) goes over into the extensively utilized approximate Wilke formula [16]

$$\mu=\sum_{i=1}^{N}\frac{x_i\mu_i'}{x_i+\sum\limits_{\substack{j=1\\ j\neq i}}^{N}x_j\sqrt{\frac{m_j}{8(m_i+m_j)}}\left[1+\left(\frac{\mu_i}{\mu_j}\right)^{1/2}\left(\frac{m_j}{m_i}\right)^{1/4}\right]^2}. \tag{2.14}$$

But if we set $A_{ij} = 5/2$, $F_{ij} = B_{ij} = 1$, then we obtain the Mason and Saxena approximation formula for gas mixtures with nearby values of the molar weights ($m_i \approx m_j$) [21]

$$\lambda=\sum_{i=1}^{N}\frac{x_i\lambda_i}{x_i+\sum\limits_{\substack{j=1\\ j\neq i}}^{N}x_j\sqrt{\frac{m_j}{8(m_i+m_j)}}\left[1+\left(\frac{\lambda_i}{\lambda_j}\right)^{1/2}\left(\frac{m_i}{m_j}\right)^{1/4}\right]^2}. \tag{2.15}$$

It is shown in [26, 27] that computations using (2.14) and (2.15) agree poorly with the results of exact computations by means of (1.1) and (1.2) for temperature values corresponding to developed dissociation and the beginning of ionization. The authors of these papers recommended the following values of A_{ij}^*, F_{ij}, and B_{ij} in the case of singly ionized air on the basis of an analysis of the collision integrals [6, 7] (let us note that H, He, and C were among the elements O, N, and E examined in [26, 27]): $A_{ij}^* = A_{ji}^* = 1.0$ for all interactions except interactions of atomic ions with their atoms of the same name for which it was recommended to set $A_{N-N^+}^* = A_{O-O^+}^* = 1.1$, $F_{ij} = F_{ji} = 1.0$ for all interactions, $B_{ij} = B_{ji} = 0.2$ for interactions of atoms and molecules with electrons (A–E, M–E), $B_{ij} = B_{ji} = 0.15$ for atoms and molecules interacting with ions (A–I, M–I), $B_{ij} = B_{ji} = 0.78$ for mutually interacting neutrals (A–A, A–M, M–M), and $B_{ij} = B_{ji} = 1.0$ for all other interactions (I–I, I–E, E–E).

For such a selection of the quantities A^*_{ij}, F_{ij}, and B_{ij} the formulas (2.11) and (2.12) yield substantially better results than the Wilke formula (2.14) [16] and the Mason–Saxena formula (2.15) [21]. At the same time our numerical computations showed that at high temperatures corresponding to the beginning of ionization, the discrepancy between the results of the exact computations in a second nonzero approximation [8] and the approximate formulas (2.11) and (2.12) can be up to ~40% for μ and ~30% for λ with the above-mentioned numerical values of the effective coefficients A^*_{ij}, F_{ij}, B_{ij}.

In this connection, the question of the possibility of obtaining more exact approximate formulas for the viscosity and transport thermal conductivity coefficients of partially ionized air which would yield a minimum deviation from the results of exact computations obtained with high approximations taken into account [8].

3. Let us note that the diagonal elements H_{ii} (1.3) and L_{ii} (2.6) can be represented in the form

$$H_{ii}=\frac{x_i^2}{\mu_i}+\sum_{\substack{k=1\\k\neq i}}^{N}\frac{2x_ix_k}{\mu_{ik}}\frac{m_k}{m_i+m_k}-\sum_{\substack{k=1\\k\neq i}}^{N}H_{ik}\approx\frac{x_i^2}{\mu_i}+\sum_{\substack{k=1\\k\neq i}}^{N}\frac{2x_ix_k}{\mu_{ik}}\frac{m_k}{m_i+m_k}; \tag{3.1}$$

$$L_{ii}=-4\frac{x_i^2}{\lambda_i}-\sum_{\substack{k=1\\k\neq i}}^{N}\frac{2x_ix_k}{(m_i+m_k)^2\lambda_{ik}A^*_{ik}}\left[\frac{15}{2}m_i^2+\frac{5}{2}m_k^2+10m_im_k\right]+$$
$$+\sum_{\substack{k=1\\k\neq i}}^{N}L_{ik}\approx-4\frac{x_i^2}{\lambda_i}-\sum_{\substack{k=1\\k\neq i}}^{N}\frac{2x_ix_k}{(m_i+m_k)^2\lambda_{ik}A^*_{ik}}\left[\frac{15}{2}m_i^2+\frac{5}{2}m_k^2+10m_im_k\right]. \tag{3.2}$$

Here the same approximation was utilized in the assumption of smallness of the nondiagonal terms $H_{ik} \ll H_{ii}$, $L_{ik} \ll L_{ii}$ $(i \neq k)$ as when obtaining the expressions (2.3) and (2.4). Substituting (3.1) into (2.3) and (3.2) into (2.4), we obtain as a result of evident manipulation

$$\mu=\sum_{i=1}^{N}\frac{\mu_ix_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N}x_kG_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}}=\sum_{i=1}^{N_n}\frac{\mu_ix_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N}x_kG_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}}+$$
$$\sum_{i=I}\frac{\mu_ix_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N}x_kG_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}}+\frac{\mu_Ex_E}{x_E+\sum\limits_{\substack{k=1\\k\neq E}}^{N}x_kG_{Ek}\sqrt{2}}; \tag{3.3}$$

$$\lambda=\sum_{i=1}^{N}\frac{\lambda_ix_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N}x_kG_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}\psi_{ik}}=\sum_{i=1}^{N_n}\frac{\lambda_ix_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N}x_kG_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}\psi_{ik}}+$$
$$+\sum_{i=I}\frac{\lambda_ix_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N}x_kG_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}\psi_{ik}}+\frac{\lambda_Ex_E}{x_E+\sum\limits_{\substack{k=1\\k\neq E}}^{N}x_kG_{Ek}\sqrt{2}\,\psi_{Ek}}, \tag{3.4}$$

where

$$G_{ik}=\frac{Q_{ik}^{(2,2)}}{Q_{ii}^{(2,2)}}; \tag{3.5}$$

$$\psi_{ik}=\frac{\frac{15}{4}m_i^2+\frac{5}{4}m_k^2+5m_im_k}{(m_i+m_k)^2A^*_{ik}}. \tag{3.6}$$

Here N_n is the number of neutral components. It follows from the expressions for the viscosity and thermal conductivity of pure gases (1.7) and (1.8) that the electron component can be neglected in multicomponent viscosity coefficient [third component in (3.3)] and the ionic component in the multicomponent transport thermal conductivity coefficient [the second component in (3.4)] since $m_E \ll m_k$ $(k \neq E)$.

In the case of a fully singly-ionized plasma in the absence of a magnetic field, the ionic viscosity μ_I (Pa·sec) and electron thermal conductivity λ_E (W/mK) are evaluated from the formulas [29]

$$\mu_I = 0.406\frac{(4\pi\varepsilon_0)^2\sqrt{m_I}(kT)^{5/2}}{e^4\ln\Lambda}; \tag{3.7}$$

$$\lambda_E = 151.44\frac{k^{7/2}T_E^{5/2}\varepsilon_0^2}{\sqrt{m_E}\,e^4\ln\Lambda_E}$$
$$\left(\Lambda = \sqrt{\frac{\varepsilon_0 k^2}{pe^2}\frac{4\pi\varepsilon_0 3kT}{\sqrt{\frac{x_I}{T^2}+\frac{x_E}{T_E^2}}}},\ \Lambda_E = \sqrt{\frac{\varepsilon_0 k^2}{pe^2}\frac{4\pi\varepsilon_0 3kT_E}{\sqrt{\frac{x_I}{T^2}+\frac{x_E}{T_E^2}}}}\right). \tag{3.8}$$

Here m_E (kg), e are the electron mass and charge, ε_0 is an electrical constant, k is the Boltzmann constant, T is the temperature of the translational degrees of freedom of the heavy components, and T_E is the electron temperature while x_I is the total ion concentration.

Computations show that for large degrees of ionization the second component in (3.3) can be replaced by the expression

$$\sum_{i=I}\frac{\mu_i x_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N} x_k G_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}} = 1.85\cdot 10^{-15}\frac{T^{5/2}}{\ln\Lambda}x_I, \tag{3.9}$$

where the value of the numerical factor [taking (3.7) into account] is selected empirically.

In sum, the following approximate formulas are assumed for evaluation of the viscosity and transport thermal conductivity coefficients of partially dissociated and ionized air:

$$\mu = \sum_{i=1}^{N_n}\frac{\mu_i x_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N} x_k G_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}} + 1.85\cdot 10^{-15}\frac{T^{5/2}}{\ln\Lambda}x_I; \tag{3.10}$$

$$\lambda = \sum_{i=1}^{N_n}\frac{\lambda_i x_i}{x_i+\sum\limits_{\substack{k=1\\k\neq i}}^{N} x_k G_{ik}\sqrt{\frac{2m_k}{m_i+m_k}}\psi_{ik}} + \frac{\lambda_E x_E}{x_E+\sum\limits_{\substack{k=1\\k\neq E}}^{N} x_k G_{Ek}\sqrt{2}\,\psi_{Ek}} + \lambda_E x_E. \tag{3.11}$$

The matrix G_{ik} [see (3.5)] is nonsymmetric, here the nondiagonal elements $G_{MM} = G_{AA} = 1.05$, $G_{AM} = 0.78$, the remaining $G_{ik} = 2.15$ (the order of the components was: O_2, N_2, NO, O, N, O^+, N^+, NO^+, O_2^+, N_2^+, E). The nondiagonal elements are converted in the evident manner

$$G_{ki} = G_{ik}\frac{Q_{ii}^{(2,2)}}{Q_{kk}^{(2,2)}} = G_{ik}\frac{\mu_k}{\mu_i}\sqrt{\frac{m_i}{m_k}} = G_{ik}\frac{\lambda_k}{\lambda_i}\sqrt{\frac{m_k}{m_i}}. \tag{3.12}$$

The matrix A_{ik}^* [see (1.10)] is symmetric: $A_{MM}^* = A_{AA}^* = 2.5$, $A_{IE}^* = 5.2$ and the remaining $A_{ik}^* = 2.3$.

It is well known that for a high degree of ionization the electrons can yield the main contribution to the transport thermal conductivity coefficient of a mixture and for good conformity with the exact computation for developed ionization in (3.11) for λ the additional term $\lambda_E x_E$ was introduced that takes account of the correction introduced into the thermal conductivity coefficient by the second nonzero approximation. Only after this was good agreement with the results of exact computations of A_{ik}^* (3) achieved successfully by variation of the coefficients A_{ik}^*. [The coefficients G_{ik} were here assumed the same as in (3.10).]

4. Values of the viscosity and transport thermal conductivity coefficients of chemically and thermodynamically ($T_E = T$) equilibrium-dissociated and ionized air are presented in Figs. 1-4 as a function of the temperature ($10^3\,\mathrm{K} \leqslant T \leqslant T^*$, T^* is the temperature corresponding to the mode of full single-ionization) for 10^2 and 10^5 Pa·sec pressures, here the solid lines are the exact computations [8], the dashes are computations by using (3.10) and (3.11) with the above-mentioned values of G_{ik}, A_{ik}^* (the dashed and solid lines in Fig. 1 are in agreement in practice in the whole temperature range under consideration), the points 1 are computations using the Armaly and Sutton formulas (2.11) and (2.12) with their recommended

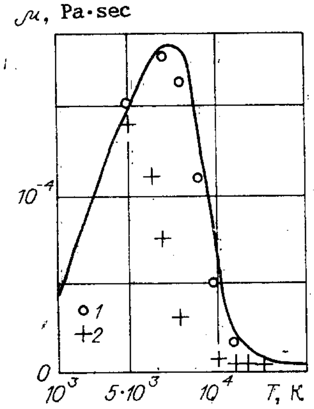

Fig. 1

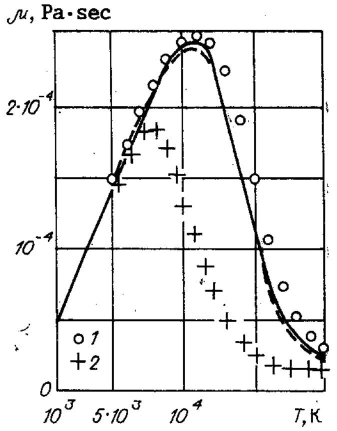

Fig. 2

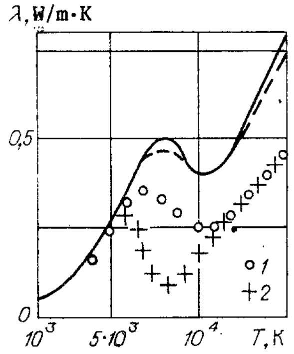

Fig. 3

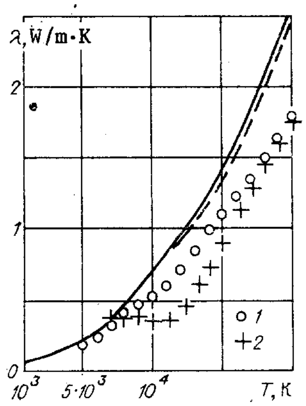

Fig. 4

values of F_{ij}, B_{ij}, and A^*_{ij} and 2 are computations using the Wilke formula (2.14) and the Mason–Saxena formula (2.15).

Computations showed that the deviation of the values of μ and λ obtained by using (3.10) and (3.11) from the exact computations is ~2% in the area of the beginning of dissociation (10^3 K $\leqslant T \leqslant 6\cdot10^3$ K). For high temperatures the discrepancy from exact computations does not exceed ~6% up to the mode of full single ionization for the pressures p = 10^2, 10^3, 10^4, 10^5, 10^6 Pa·sec. It is seen from the figures that the Wilke and Mason–Saxena formulas yield an ~50-70% error for developed dissociation. The Armaly and Sutton formulas are substantially more accurate, however, for a significant degree of ionization the deviation from the exact computation can be up to ~40% for μ and ~30% for λ. Computations by the Wilke and Mason–Saxena formulas were in good agreement with the exact computations just for relatively low temperatures. The Wilke and Mason–Saxena formulas were proposed in their time for computations of the properties of nondissociated gases and were confirmed in inert gas mixtures. Consequently, their interpolation in the high temperature range yields a large error. Armaly and Sutton relied on computations [6] that differ by 10% from the exact [8] for developed dissociation and by ~15% (μ) and ~60% (λ) for full dissociation and developed ionization because of utilization of the lowest approximations. But even with the data of [6] the computations using the Armaly and Sutton formulas diverge by more than 10%.

Therefore, on the basis of exact computations simple semiempirical formulas are obtained for determination of the viscosity and transport thermal conductivity coefficients (due to energy transfer by translational degrees of freedom of the components) of a mixture that permits diminution of the machine time expenditure for the solution of complex gasdynamic problems. The discrepancy from the results of exact computations is here ~1-2% in the low temperature range, not more than 3% in a broad range of temperatures and pressures and only reaches 6% in a narrow transitional domain. The form of the expressions (3.10) and (3.11), the independence of the given approximation coefficients G_{ik} and A^*_{ik} on the temperature and pressure as well as the good correspondence with exact numerical computations for the pressure values in an interval of several orders, yields a foundation for assuming that (3.10) and (3.11) can be used with good accuracy for an arbitrary chemical composition of dissociated and ionized air.

Separate investigations are evidently required for other gas mixtures. However, the approach proposed by Armaly and Sutton and developd in this paper can apparently be used for a sufficiently broad class of gas and plasma mixtures. Briefly, the main ideas of this approach can be formulated as follows: 1) expressions for μ and λ of the gas mixtures in a first nonzero approximation are taken as basis; 2) the quantities H_{ij}, L_{ij} $(i \neq j)$ are neglected everywhere in these expressions; 3) terms taking account of the electron viscosity and ion thermal conductivity are discarded in the formulas obtained; 4) by using asymptotic expressions for μ and λ [29] the emergence at the exact value "is shortened" in the case of full single ionization; 5) quantities slightly dependent on the temperature and pressure are extracted, and replaced by effective numerical coefficients that are selected empirically from a comparison with the results of exact computations.

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RESISTANCE OF A BODY WITH AN INTRINSIC MAGNETIC FIELD IN A SUPERSONIC FLOW OF A PARTIALLY IONIZED GAS

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The presence of an intrinsic magnetic field substantially alters the nature of the flow, the structure of the perturbed zone, and the charged particle distribution of a body surface in supersonic rarefied plasma flow [1]. A system of currents or permanent magnets can be the source of the magnetic field of the body. The decisive influence of the self-consistent field on the charged particle distribution in the neighborhood of bodies with an intrinsic magnetic field is clarified in [1, 2] for $\rho_e \ll R \ll \rho_i$ (R is the characteristic body dimension, and ρ_α is the Larmor radius of particles of the species α). The perturbations induced by the intrinsic magnetic field result in a change in the functional characteristics of different systems and singularities of dynamic body interaction with the flow. Results of an approximate numerical solution of the problem of MHD-interaction of a body with a supersonic rarefied plasma flow [3, 4] indicate the possibility of controlling the forces acting on the body, the change in heat elimination to the surface. Experimental data are scarce, limited to a narrow band of interaction parameters, and do not take account of the influence of the body surface geometry [5, 6]. Results of an experimental investigation of the influence of the intrinsic magnetic field on resistance of bodies of simple geometric shape (disc, sphere, cylinder, cone) are presented in this paper. Dependences of the body frontal drag coefficient on the magnetic field intensity are determined for $\mathbf{U}_\infty \parallel \mathbf{H}$ and $\mathbf{U}_\infty \perp \mathbf{H}$ ($\mathbf{U}_\infty$, and $\mathbf{H}$ are the flow velocity and magnetic field intensity vectors). The possibility of an effective MHD deceleration of bodies in a supersonic rarefied plasma flow is given a foundation.

1. Experiments were performed in a plasma gasdynamic installation in a partially ionized nitrogen flow generated by a gas-discharge accelerator with ionization of the working body by electron impact and plasma "self-acceleration." The diagram of the installation is presented, in principle, in [7]. An accelerated plasma flow entered the working chamber, in which the residual gas pressure was $\sim 10^{-5}$ Pa. Evacuation was realized by a vacuum electrodischarge unit of AVED-40/800M type and a turbomolecular pump of TMH-500 type. The rarefied plasma flow parameters at the $\sim 10^{-1}$-10^{-3} Pa working pressures in the chamber were measured by using mobile electrical probes and a multielectrode probe analyzer. Measurement of the current–voltage characteristics and the derivatives of the probe current was performed in the automatic mode. The scheme of probe measurements with current–voltage characteristics

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